Random dimensionality reduction and sparse recovery algorithms

Massimo Fornasier

We start by presenting the optimal dimensionality and complexity properties of an abstract coding-decoding system. The coding of high-dimensional vectors in $\mathbb{R}^N$ is performed by means of a linear map into a lower dimensional space $\mathbb{R}^m$, where $m \ll N$, and the decoding is performed by means of any nonlinear map with an error proportional to the best k-term approximation. Then we will show that such an ideal and optimal coding-decoding system can be actually realized in practice by compressing high-dimensional vectors into lower dimension via suitable *random matrix embedding* and recovering them by convex optimization, *minimizing the* ℓ_1^N*-norm* of competitors. We exemplarily present an algorithm to perform efficiently the latter convex optimization, based on iteratively reweighted least squares. We eventually illustrate three variations on the theme, related to applications beyond coding-decoding. First we address uniform approximation algorithms with polynomial complexity to recover high-dimensional functions from random sampling, breaking the curse of dimensionality. Then we propose randomized algorithms to simulate high-dimensional dynamical systems modeling complex interacting agents via multiple projections in lower dimension. We conclude by presenting very recent results in the sparse stabilization and optimal control of such dynamical systems to enforce pattern formation.