

# Dynamic utilities and long term interest rates

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A large debate is open for several years within mathematical finance about the criterion to optimize. A example is given by the forward dynamic utilities introduced by M. Musiela and T. Zariphopoulou in (2003-2010) to model possible changes over the time of individual preferences of an agent. In particular, there is no-prespecified trading horizon. These point of view applied to consumption problem is of large importance in long term decision making. We characterize these dynamic utilities in terms of Hamilton Jacoby Bellman SPEs. An explicit solution is given using monotonic solutions of two stochastic differential equations and their inverse, with a nice interpretation in terms of optimal wealth when these utilities satisfy a property of consistency with a given incomplete financial market.

In the controversy on the discount rate used in financing long term projects, such a criterion leads to a yield curve depending of the wealth of the economy.